Supplementary Materials for Medical image segmentation via single-source domain generalization with random amplitude spectrum synthesis*

Qiang Qiao[1,2], Wenyu Wang[1,2], Meixia Qu[1,2], Kun Su[1,2], Bin Jiang[1,2(✉)], and Qiang Guo[3,4(✉)]

[1] School of Mechanical, Electrical & Information Engineering, Shandong University, Weihai, China
jiangbin@sdu.edu.cn
[2] Shenzhen Research Institute of Shandong University, Shenzhen, China
[3] School of Computer Science and Technology, Shandong University of Finance and Economics, Jinan, China
guoqiang@sdufe.edu.cn
[4] Shandong Provincial Key Laboratory of Digital Media Technology, Jinan, China

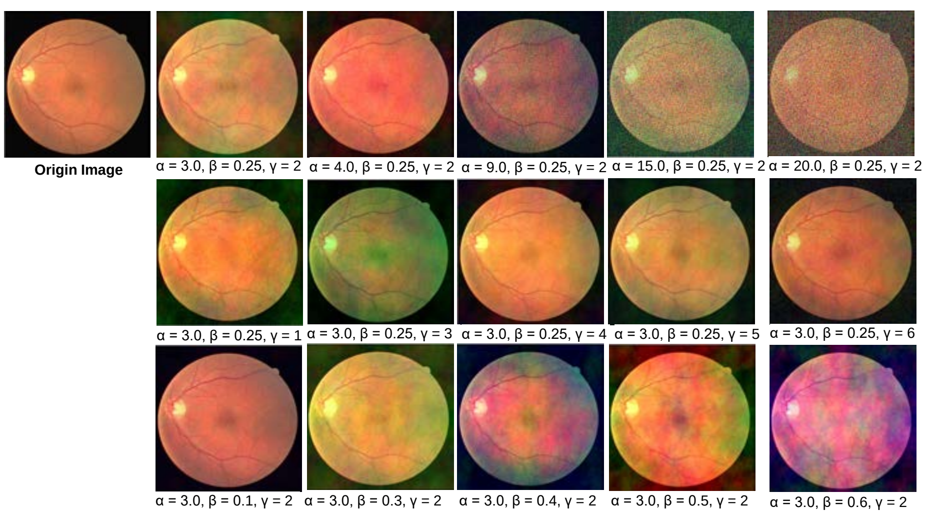

Fig. 1. Visualisation of different hyperparameter settings in RASS.

Algorithm 1 RASS for 3D and 2D Medical Image

Input: $\boldsymbol{x_s}$ ▷ 2D or 3D medical image
RASS parameters: α, β, γ ▷ Set values
if $\dim(\boldsymbol{x_s}) == 3$ **then** ▷ Process as 3D image
  $\mathcal{F}(\boldsymbol{x_s}) \leftarrow \text{FFT3D}(\boldsymbol{x_s})$ ▷ Obtain 3D Fourier spectrum
  $\mathcal{A}(\boldsymbol{x_s}), \mathcal{P}(\boldsymbol{x_s}) \leftarrow \text{Abs}(\mathcal{F}(\boldsymbol{x_s}), \text{Ang}(\mathcal{F}(\boldsymbol{x_s}))$ ▷ Amplitude and phase spectrum
  $\sigma_{H\times W\times D} \leftarrow \text{Meshgrid}([-H/2, H/2], [-W/2, W/2], [-D/2, D/2])$
  for $m \in [-H/2, H/2]$ **do**
    for $n \in [-W/2, W/2]$ **do**
      for $p \in [-D/2, D/2]$ **do**
        $\sigma[m,n,p] \leftarrow \left(2\alpha\sqrt{\frac{m^2+n^2+p^2}{H^2+W^2+D^2}}\right)^{\gamma} + \beta$ ▷ Calculate perturbation $\boldsymbol{\sigma}$
  $\boldsymbol{\delta}_{H\times W\times D} \sim \mathcal{N}(1, \boldsymbol{\sigma^2}_{H\times W\times D})$ ▷ Sample
  $\mathcal{A}(\boldsymbol{x_s}) \leftarrow \text{FFTShift}(\mathcal{A}(\boldsymbol{x_s}))$
  $\tilde{\mathcal{A}}(\boldsymbol{x_s}) \leftarrow \boldsymbol{\delta}_{H\times W\times D} \odot \mathcal{A}(\boldsymbol{x_s})$ ▷ Synthesize amplitude spectrum
  $\tilde{\mathbf{x}} \leftarrow \text{Inverse-FFT3D}(\tilde{\mathcal{A}}(\boldsymbol{x_s}), \mathcal{P}(\boldsymbol{x_s}))$ ▷ Recover the image
else if $\dim(\boldsymbol{x_s}) == 2$ **then** ▷ Process as 2D image
  $\mathcal{F}(\boldsymbol{x_s}) \leftarrow \text{FFT2D}(\boldsymbol{x_s})$ ▷ Obtain 2D Fourier spectrum
  $\mathcal{A}(\boldsymbol{x_s}), \mathcal{P}(\boldsymbol{x_s}) \leftarrow \text{Abs}(\mathcal{F}(\boldsymbol{x_s})), \text{Ang}(\mathcal{F}(\boldsymbol{x_s}))$ ▷ Amplitude and phase spectrum
  $\sigma_{H\times W} \leftarrow \text{Meshgrid}(-H/2, H/2, -W/2, W/2)$
  for $m \in [-H/2, H/2]$ **do**
    for $n \in [-W/2, W/2]$ **do**
      $\sigma[m,n] \leftarrow \left(2\alpha\sqrt{\frac{m^2+n^2}{H^2+W^2}}\right)^{\gamma} + \beta$ ▷ Calculate perturbation $\boldsymbol{\sigma}$ for 2D
  $\boldsymbol{\delta}_{H\times W} \sim \mathcal{N}(1, \boldsymbol{\sigma^2}_{H\times W})$ ▷ Sample
  $\mathcal{A}(\boldsymbol{x_s}) \leftarrow \text{FFTShift}(\mathcal{A}(\boldsymbol{x_s}))$
  $\tilde{\mathcal{A}}(\boldsymbol{x_s}) \leftarrow \boldsymbol{\delta}_{H\times W} \odot \mathcal{A}(\boldsymbol{x_s})$ ▷ Synthesize amplitude spectrum
  $\tilde{\mathbf{x}} \leftarrow \text{Inverse-FFT2D}(\tilde{\mathcal{A}}(\boldsymbol{x_s}), \mathcal{P}(\boldsymbol{x_s}))$ ▷ Recover the image
end if

Table 1. Ablation study on different backbone.

Backbone	FeTA2021	IOSTAR	LES-AV
U-Net	76.03.$_{\pm 0.36}$	65.33$_{\pm 0.19}$	72.33$_{\pm 0.17}$
MedNeXt	76.34.$_{\pm 0.19}$	65.79$_{\pm 0.21}$	72.83$_{\pm 0.19}$
SegResNet	**76.56$_{\pm 0.23}$**	**65.86$_{\pm 0.12}$**	**72.88$_{\pm 0.07}$**

Table 2. Ablation study of RASS on the FeTA2021 dataset. γ is fixed to 2.0.

parameter	$\beta = 0.15$	$\beta = 0.25$	$\beta = 0.45$
$\alpha = 2.0$	76.51$_{\pm 0.31}$	76.32$_{\pm 0.32}$	76.16$_{\pm 0.31}$
$\alpha = 3.0$	76.52$_{\pm 0.28}$	**76.56$_{\pm 0.23}$**	76.08$_{\pm 0.26}$
$\alpha = 9.0$	75.52$_{\pm 0.29}$	75.67$_{\pm 0.25}$	75.31$_{\pm 0.33}$

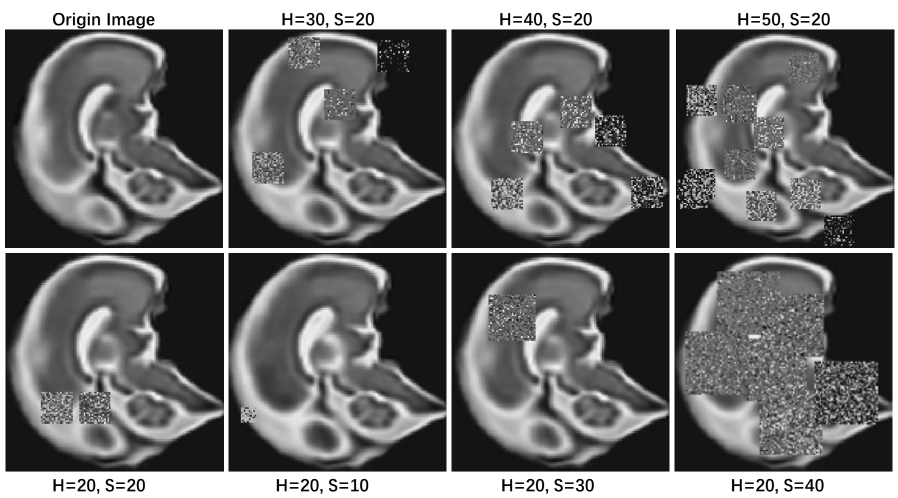

Fig. 2. Visualization of images after different mask sizes and numbers in RMS.