

## A Appendix

### A.1 Non-Contrastive VicReg Loss

VICReg loss with *invariance*, *variance*, and *covariance* terms, is as follows:

$$\begin{aligned} \ell_{\text{contr}}(Z_t, Z_{t+\Delta t}) &= \lambda_S \cdot S(Z_t, Z_{t+\Delta t}) + \lambda_V \cdot (V(Z_t) + V(Z_{t+\Delta t})) \\ &\quad + \lambda_C \cdot (C(Z_t) + C(Z_{t+\Delta t})) \end{aligned} \quad (6)$$

$$S(Z, Z') = \frac{1}{n} \sum_i \|z_i - z'_i\|_2^2 \quad V(Z) = \frac{1}{d} \sum_{j=1}^d \max(0, 1 - \text{std}(z^j, \epsilon))$$

$$C(Z) = \frac{1}{d} \sum_{i \neq j} [\text{Cov}(Z)]_{i,j}^2,$$

$$\text{where } \text{Cov}(Z) = \frac{1}{n-1} \sum_{i=1}^n (z_i - \bar{z})(z_i - \bar{z})^T.$$

$\lambda_S$ ,  $\lambda_V$ , and  $\lambda_C$  are set to 15, 25, 5 to bring their magnitude in the same range.

### A.2 Derivation of Displacement Map Regularization Term

The pairwise RankNet [4] loss is based on cross-entropy loss for calculating the score-based ranking a pair of elements. Let  $f$  be a scoring function for the items  $x_i$  and  $x_j$ , such that their respective scores are  $s_i = f(x_i)$  and  $s_j = f(x_j)$ . Concordantly the score difference is defined as  $s_{ij} = s_i - s_j$ . Then, the probability of ranking  $x_i$  greater than  $x_j$  is defined using the logistic function:

$$\mathcal{P}_{ij} = \frac{e^{s_{ij}}}{1 + e^{s_{ij}}} = \frac{1}{1 + e^{-s_{ij}}} \quad (7)$$

When ranking the items, there are 3 values for ground truth  $Y$ ; 1 when  $x_i$  has higher rank than  $x_j$ , 0 when the relation is reversed, and  $\frac{1}{2}$  when both items have the same rank. Accordingly, cross-entropy loss for correctly ranking the items is:

$$\mathcal{L}_{ce} = -Y \cdot \log(\mathcal{P}_{ij}) - (1 - Y) \cdot \log(1 - \mathcal{P}_{ij}) \quad (8)$$

In TC,  $s_i$  becomes  $r_{t+\Delta t}$  and  $s_j$  becomes  $r_t$ , hence  $s_{ij}$  is the DM. For the cross-entropy loss, we implemented  $s_{ij}$  as the norm of the DM. Also the ranking class is always 1 because the time difference is kept positive, thus the Eq. 8 becomes:

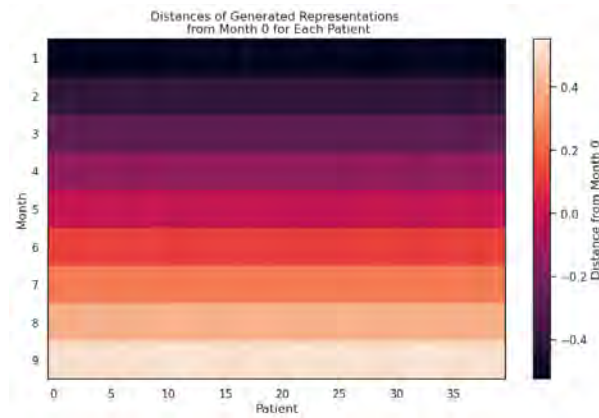
$$\begin{aligned} \mathcal{L}_{reg} &= -1 \cdot \log(\mathcal{P}_{ij}) - (0) \cdot \log(1 - \mathcal{P}_{ij}) = -\log\left(\frac{1}{1 + e^{-s_{ij}}}\right) \\ &= \log(1 + e^{-s_{ij}}) = \log(1 + \exp(-\|\rho_\psi(r_t, \Delta t)\|_2)) \end{aligned} \quad (9)$$

**Table 2.** Contrastive Augmentations and OCT Details

Contrastive Transformations	Parameter
Random Crop & Resize (percentage)	0.4 - 0.8
Random Horizontal Flip (probability)	0.5
Random Color Jittering (probability)	0.8
Random Gaussian Blur (kernel size)	21
Random Solarize (threshold)	0.5
Random Rotation (degrees)	$\pm 5$
Random Translation (percentage)	$\pm 0.05$
Input Time Difference	1-12 Months
Normalization Mean & std	(0.202, 0.113)
OCT Scanner	Cirrus OCT
Resolution	$6 \times 6 \times 2 \text{ mm}^3$
Projector MLP dimensions	4096-4096-4096
Predictor $\rho_\psi$ MLP dimensions	2049-2048
GPU Used	Nvidia A100 80GB

**Table 3.** Computational Costs of Each Model

Model	Batch Update (Seconds)	# Trainable Parameters
VICReg	9.3	65.4M
ESSL	16.3	78.0M
AugSelf	14.2	78.0M
EquiMod	12.6	124.2M
TC	10.4	82.2M

**Fig. 3.** Distance rankings between the original representation  $r_0$  and its propoageted prediction  $r'_{t+\Delta t}$  for 9 consecutive months for 40 patients.