Quality-Aware Fuzzy Min-Max Neural Networks for Dynamic Brain Network Analysis

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1 Supplementary Material

To identify all overlapping regions of hyperboxes of different classes for dealing with the fuzzy information, we employ nine cases as hyperbox overlap test rules. Hyperbox overlap test rules are as follows:

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 \begin{array}{ll} 1. & BV_{ji}^{v} < BV_{ki}^{v} < BW_{ji}^{v} < BW_{ki}^{v}, \delta^{\mathrm{new}} = \min(BW_{ji}^{v} - BV_{ki}^{v}, \delta^{\mathrm{old}}); \\ 2. & BV_{ki}^{v} < BV_{ji}^{v} < BW_{ki}^{v} < BW_{ji}^{v}, \delta^{\mathrm{new}} = \min(BW_{ki}^{v} - BV_{ji}^{v}, \delta^{\mathrm{old}}); \\ 3. & BV_{ji}^{v} = BV_{ki}^{v} < BW_{ji}^{v} < BW_{ki}^{v}, \\ \delta^{\mathrm{new}} = \min(\min(BW_{ji}^{v} - BV_{ki}^{v}, BW_{ki}^{v} - BV_{ji}^{v}), \delta^{\mathrm{old}}); \\ 4. & BV_{ji}^{v} < BV_{ki}^{v} < BW_{ji}^{v} = BW_{ki}^{v}, \\ \delta^{\mathrm{new}} = \min(\min(BW_{ji}^{v} - BV_{ki}^{v}, BW_{ki}^{v} - BV_{ji}^{v}), \delta^{\mathrm{old}}); \\ 5. & BV_{ki}^{v} = BV_{ji}^{v} < BW_{ki}^{v} < BW_{ji}^{v}, \\ \delta^{\mathrm{new}} = \min(\min(BW_{ji}^{v} - BV_{ki}^{v}, BW_{ki}^{v} - BV_{ji}^{v}), \delta^{\mathrm{old}}); \\ 6. & BV_{ki}^{v} < BV_{ji}^{v} < BW_{ki}^{v} = BW_{ji}^{v}, \\ \delta^{\mathrm{new}} = \min(\min(BW_{ji}^{v} - BV_{ki}^{v}, BW_{ki}^{v} - BV_{ji}^{v}), \delta^{\mathrm{old}}); \\ 7. & BV_{ji}^{v} < BV_{ki}^{v} \leq BW_{ki}^{v} < BW_{ji}^{v}, \\ \delta^{\mathrm{new}} = \min(\min(BW_{ji}^{v} - BV_{ki}^{v}, BW_{ki}^{v} - BV_{ji}^{v}), \delta^{\mathrm{old}}); \\ 8. & BV_{ki}^{v} < BV_{ji}^{v} \leq BW_{ji}^{v} < BW_{ki}^{v}, \\ \delta^{\mathrm{new}} = \min(\min(BW_{ji}^{v} - BV_{ki}^{v}, BW_{ki}^{v} - BV_{ji}^{v}), \delta^{\mathrm{old}}); \\ 9. & BV_{ki}^{v} = BV_{ji}^{v} < BW_{ki}^{v} = BW_{ji}^{v}, \\ \delta^{\mathrm{new}} = \min(BW_{ki}^{v} - BV_{ji}^{v}, \delta^{\mathrm{old}}). \end{array}
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Initial set $\delta^{\rm old}=1$, an overlapping region is observed if $\delta^{\rm old}-\delta^{\rm new}<1$ for each dimension. Then, set $\Delta=i$ and $\delta^{old}=\delta^{new}$ to check the next dimension. Hyperbox overlap test will cease when $\delta^{\rm old}-\delta^{\rm new}=1$.

The contraction rules are developed based on the nine cases of the hyperbox overlap test. All cases are examined to determine a proper adjustment. Hyperbox contraction rules are as follows:

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 \begin{split} &1. \;\; BV^v_{j\Delta} < BV^v_{k\Delta} < BW^v_{j\Delta} < BW^v_{k\Delta}, \\ &\;\; (BW^v_{j\Delta})^{\mathrm{new}} = (BV^v_{k\Delta})^{\mathrm{new}} = ((BW^v_{j\Delta})^{\mathrm{old}} + (BV^v_{k\Delta})^{\mathrm{old}})/2; \\ &2. \;\; BV^v_{k\Delta} < BV^v_{j\Delta} < BW^v_{k\Delta} < BW^v_{j\Delta}, \\ &\;\; (BW^v_{k\Delta})^{\mathrm{new}} = (BV^v_{j\Delta})^{\mathrm{new}} = ((BW^v_{k\Delta})^{\mathrm{old}} + (BV^v_{j\Delta})^{\mathrm{old}})/2; \end{split}
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- 3. $BV_{j\Delta}^v = BV_{k\Delta}^v < BW_{j\Delta}^v < BW_{k\Delta}^v, (BV_{k\Delta}^v)^{\text{new}} = (BW_{j\Delta}^v)^{\text{old}};$
- 4. $BV_{i\Delta}^{v} < BV_{k\Delta}^{v} < BW_{i\Delta}^{v} = BW_{k\Delta}^{v}, (BW_{i\Delta}^{v})^{\text{new}} = (BV_{k\Delta}^{v})^{\text{old}};$
- 5. $BV_{k\Delta}^v = BV_{j\Delta}^v < BW_{k\Delta}^v < BW_{j\Delta}^v, (BV_{j\Delta}^v)^{\text{new}} = (BW_{k\Delta}^v)^{\text{old}};$
- 6. $BV_{k\Delta}^{v} < BV_{j\Delta}^{v} < BW_{k\Delta}^{v} = BW_{j\Delta}^{v}, (BW_{k\Delta}^{v})^{\text{new}} = (BV_{j\Delta}^{v})^{\text{old}};$ 7. $BV_{j\Delta}^{v} < BV_{k\Delta}^{v} \le BW_{k\Delta}^{v} < BW_{j\Delta}^{v}, (BW_{k\Delta}^{v} BV_{j\Delta}^{v}) < (BW_{j\Delta}^{v} BV_{k\Delta}^{v}),$ $(BV_{j\Delta}^v)^{\text{new}} = (BW_{k\Delta}^v)^{\text{old}};$
- 8. $BV_{j\Delta}^v < BV_{k\Delta}^v \le BW_{k\Delta}^v < BW_{j\Delta}^v, (BW_{k\Delta}^v BV_{j\Delta}^v) > (BW_{j\Delta}^v BV_{k\Delta}^v),$ $(B\widetilde{W}_{j\Delta}^{v})^{\text{new}} = (BV_{k\Delta}^{v})^{\text{old}};$
- 9. $BV_{k\Delta}^{vJ} < BV_{j\Delta}^{v} \le B\widetilde{W}_{j\Delta}^{v} < BW_{k\Delta}^{v}, (BW_{k\Delta}^{v} BV_{j\Delta}^{v}) < (BW_{j\Delta}^{v} BV_{k\Delta}^{v}),$ $(BW_{k\Delta}^v)^{\text{new}} = (BV_{i\Delta}^v)^{\text{old}};$
- $10. \ BV_{k\Delta}^{v} < BV_{j\Delta}^{v} \le BW_{j\Delta}^{v} < BW_{k\Delta}^{v}, (BW_{k\Delta}^{v} BV_{j\Delta}^{v}) > (BW_{j\Delta}^{v} BV_{k\Delta}^{v}),$ $(BV_{k\Delta}^{v})^{\text{new}} = (BW_{j\Delta}^{v})^{\text{old}};$
- 11. $BV_{j\Delta}^{v} = BV_{k\Delta}^{v} < BW_{j\Delta}^{v} = BW_{k\Delta}^{v}$ $(BW_{i\Delta}^v)^{\text{new}} = (BV_{k\Delta}^v)^{\text{new}} = ((BW_{i\Delta}^v)^{\text{old}} + (BV_{k\Delta}^v)^{\text{old}})/2;$
- 12. $BV_{k\Delta}^{vJ} = BV_{j\Delta}^{v} < BW_{k\Delta}^{v} = BW_{j\Delta}^{v},$ $(BW_{k\Delta}^v)^{\text{new}} = (BV_{j\Delta}^v)^{\text{new}} = ((BW_{k\Delta}^v)^{\text{old}} + (BV_{j\Delta}^v)^{\text{old}})/2.$