Proposition 1. *Consider a set of multivectors $x^i \in \mathbf{G}(3,0,1)$ and denote by*

$$t(x^i) := \frac{1}{x^i_{123}} \begin{pmatrix} x^i_{012} \\ x^i_{013} \\ x^i_{023} \end{pmatrix}$$

the extraction of point coordinates from a multivector. Assume $x^i_{123} > 0$. Then the point extracted from convex combination of x^i is an element of the convex hull of $\{t(x^i)\}_i$.

Proof. Let $w = \sum_i \omega_i x^i$, such that $\omega_i > 0$ and $\sum_i \omega_i = 1$. Then

$$t(w) = \frac{1}{w_{123}} \begin{pmatrix} w_{012} \\ w_{013} \\ w_{023} \end{pmatrix} = \frac{1}{\sum_i \omega_i x^i_{123}} \begin{pmatrix} \sum_i \omega_i x^i_{012} \\ \sum_i \omega_i x^i_{013} \\ \sum_i \omega_i x^i_{023} \end{pmatrix}.$$

Define

$$\omega'_i = \frac{\omega_i x^i_{123}}{\sum_j \omega_j x^j_{123}}$$

and note that $\omega'_i > 0$ and $\sum_i \omega'_i = 1$. Now

$$\begin{pmatrix} \sum_i \frac{\omega_i}{\sum_j \omega_j x^j_{123}} x^i_{012} \\ \sum_i \frac{\omega_i}{\sum_j \omega_j x^j_{123}} x^i_{013} \\ \sum_i \frac{\omega_i}{\sum_j \omega_j x^j_{123}} x^i_{023} \end{pmatrix} = \begin{pmatrix} \sum_i \frac{\omega'_i}{x^i_{123}} x^i_{012} \\ \sum_i \frac{\omega'_i}{x^i_{123}} x^i_{013} \\ \sum_i \frac{\omega'_i}{x^i_{123}} x^i_{023} \end{pmatrix}$$
$$= \sum_i \omega'_i t(x^i)$$

and thus

$$t(w) = t(\sum_i \omega_i x^i) = \sum_i \omega'_i t(x^i).$$

Since the convex hull of a set of points is defined as the set of all convex combinations of its elements, $t(\sum_i \omega_i x^i)$ is an element of the convex hull of $\{t(x^i)\}_i$. □

On the choice of sub-sampling ratio. We sub-sampled the coronary volume meshes aggressively to make LaB-GATr fit on an NVIDIA L40 (48 GB). We sub-sample the cortical meshes to the same number of tokens. We increased the sampling ratio for the coronary surface meshes while still affording the same batch size used by GATr.