—Supplementary Material— BPaCo: Balanced Parametric Contrastive Learning for Long-tailed Medical Image Classification

Zhiyuan Cai, Tianyunxi Wei, Li Lin, Hao Chen, Xiaoying Tang

Department of Electronic and Electrical Engineering,
Southern University of Science and Technology, Shenzhen, China
tangxy@sustech.edu.cn
[2] Department of Computer Science Engineering,
The Hong Kong University of Science and Technology, Hong Kong SAR, China
jhc@ust.hk
[3] Department of Electrical and Electronic Engineering,
The University of Hong Kong, Hong Kong SAR, China
[4] Jiaxing Research Institute, Southern University of Science and Technology,
Jiaxing, China

Proof on Theorem 1

$$\begin{aligned}\mathcal{L}_{ave}(Z;Y,A,y) &= -\sum_{i\in A_y}\frac{1}{|A_y|-1}\sum_{p\in\{A_y\setminus\{i\}\}}\log\frac{\exp\left(z_p\cdot z_i\right)}{\sum_{j\in\mathcal{Y}}\frac{1}{|A_j|}\sum_{k\in A_j}\exp\left(z_k\cdot z_i\right)}\\ &= \sum_{i\in A_y}\log\left(\frac{\sum_{j\in\mathcal{Y}_A}\frac{1}{|A_j|}\sum_{k\in A_j}\exp\left(z_i\cdot z_k\right)}{\prod_{p\in A_y\setminus\{i\}}\exp\left(z_i,z_p\right)^{\frac{1}{|A_y|-1}}}\right)\\ &= \sum_{i\in A_y}\log\left(\frac{\sum_{j\in\mathcal{Y}_A}\frac{1}{|A_j|}\sum_{k\in A_j}\exp\left(z_i\cdot z_k\right)}{\exp\left(\frac{1}{|A_y|-1}\sum_{p\in A_y\setminus\{i\}}z_i\cdot z_p\right)}\right).\end{aligned} \tag{1}$$

By applying Jensen's inequality, we have

$$\begin{aligned}&\frac{1}{|A_y|-1}\sum_{k\in A_y\setminus\{i\}}\exp\left(z_i\cdot z_k\right)\geq\exp\left(\frac{1}{|A_y|-1}\sum_{k\in A_y\setminus\{i\}}z_i\cdot z_k\right)\\ &\frac{1}{|A_j|}\sum_{k\in A_j,j\neq y}\exp\left(z_i\cdot z_k\right)\geq\exp\left(\frac{1}{|A_j|}\sum_{k\in A_j}z_i\cdot z_k\right).\end{aligned} \tag{2}$$

Thus, the sum can be written as follows

$$
\begin{aligned}
&\sum_{j\in\mathcal{Y}_A, j\neq y} \exp\left(\frac{1}{|A_j|}\sum_{k\in A_j}(z_i\cdot z_k)\right) \geq \\
&\exp\left(\frac{1}{|A_y-1|}\sum_{p\in A_y\setminus\{i\}} z_i\cdot z_p\right) + \sum_{j\in\mathcal{Y}_A, j\neq y}\exp\left(\frac{1}{|A_j|}\sum_{k\in A_j} z_i\cdot z_k\right).
\end{aligned} \tag{3}
$$

By applying Jensen's inequality again on the latter term, we obtain

$$
\begin{aligned}
&\sum_{j\in\mathcal{Y}_A, j\neq y} \exp\left(\frac{1}{|A_j|}\sum_{k\in A_j}(z_i\cdot z_k)\right) \geq \\
&(|\mathcal{Y}_B|-1)\exp\left(\frac{1}{|\mathcal{Y}_B|-1}\sum_{j\in\mathcal{Y}_B, j\neq y}\frac{1}{|B_j|}\sum_{k\in B_j} z_i\cdot z_k\right).
\end{aligned} \tag{4}
$$

Thus, for a specific mini-batch and *queue*, Eq. 3 holds.

Proof on Theorem 2
Firstly

$$
\begin{aligned}
\mathcal{L}_{BPaCo}(Z;Y) &= \sum_{A\in\mathcal{A}}\sum_{y\in\mathcal{Y}}\mathcal{L}_{BPaCo}(Z;Y,A,y) \\
&\geq \sum_{A\in\mathcal{A}}\sum_{y\in\mathcal{Y}}\sum_{i\in A_y}\log(1+(|\mathcal{Y}|-1)\exp(S(Z;Y,A,y))).
\end{aligned} \tag{5}
$$

Then

$$
\mathcal{L}_{BPaCo}(Z;Y) \geq |\mathcal{D}| \log\left(1+(|\mathcal{Y}|-1)\exp\left(\sum_{A\in\mathcal{A}}\sum_{y\in\mathcal{Y}}\sum_{i\in A_y} S(Z;Y,A,y)\right)\right). \tag{6}
$$

As illustrated in [10], we can rewrite $\mathcal{L}_{BPaCo}$ as

$$
\begin{aligned}
\mathcal{L}_{BPaCo} &= \sum_{A\in\mathcal{A}}\sum_{y\in\mathcal{Y}}\sum_{i\in A_y}\mathcal{L}_i, \\
\mathcal{L}_i &= -\log\frac{\exp\left(z_i\cdot z_{c_y}\right)}{\exp\left(z_i\cdot z_{c_y}\right)+\sum_{j\in\mathcal{Y}\setminus\{y\}}\exp\left(z_i\cdot z_{c_j}\right)}, \\
\mathcal{L}_i &\geq \log\left(1+(K-1)\exp\left(-\frac{K}{K-1}\right)\right).
\end{aligned} \tag{7}
$$

Finally, Eq. 5 holds.

Table A1. Details of the involved diseases in ISIC2018, APTOS2019 and OCTA500.

ISIC2018		APTOS2019		OCTA500	
NV	Melanocytic Nevus	No DR	Without Diabetic Retinopathy	Normal	Normal
MEL	Melanoma	Moderate	Moderate Diabetic Retinopathy	OTHERS	Diseased samples < 8
BKL	Benign Keratosis	Mild	Mild Diabetic Retinopathy	DR	Diabetic Retinopathy
BCC	Basal Cell Carcinoma	Proliferative	Proliferative Diabetic Retinopathy	AMD	Age-related Macular Degeneration
AKIEC	Actinic Keratosis	Severe	Severe Diabetic Retinopathy	CNV	Choroidal Neovascularization
VASC	Vascular Lesion	-	-	CSC	Central Serous Chorioretinopathy
DF	Dermatofibroma	-	-	RVO	Retinal Vein Occlusion