Supplementary material: Hierarchical multiple instance learning for COPD grading with relatively specific similarity

Hao Zhang[1,3], Mingyue Zhao[2,3], Mingzhu Liu[6], Jiejun Luo[6], Yu Guan[7], Jin Zhang[7], Yi Xia[7], Di Zhang[7], Xiuxiu Zhou[7], Li Fan[7], Shiyuan Liu[7], and S. Kevin Zhou[2,3,4,5]

[1] School of Computer Science and Technology, University of Science and Technology of China
[2] School of Biomedical Engineering, Division of Life Sciences and Medicine, University of Science and Technology of China (USTC), Hefei Anhui, 230026, China
[3] Center for Medical Imaging, Robotics, Analytic Computing & Learning (MIRACLE), Suzhou Institute for Advance Research, USTC, Suzhou Jiangsu, 215123, China
[4] Key Laboratory of Precision and Intelligent Chemistry, USTC, Hefei Anhui,230026, China
[5] Key Laboratory of Intelligent Information Processing of Chinese Academy of Sciences (CAS), Institute of Computing Technology, CAS
[6] Department of Radiology, Tongji Hospital of Tongji University
[7] Department of Radiology, the Second Affiliated Hospital of PLA Naval Medical University

1 Absolutely Abstract Similarity (AAS)

The AAS should satisfy the following constraints:

1. $n \to \infty$, there are countless classes in the abstract world, so the size of this adjacency matrix is infinite.
2. $AAS_{i,j} \equiv AAS_{j,i}$

   Proof.

$$AAS_{i,j} = \lim_{n\to\infty} \frac{\gamma(I,J)}{\sum_{j=1}^{n} \gamma(\beta,J)} = \lim_{n\to\infty} \frac{\gamma(I,J)}{n\mu},$$

$$AAS_{j,i} = \lim_{n\to\infty} \frac{\gamma(J,I)}{n\mu} = AAS_{i,j}.$$

3. $AAS > 0$, in the real world, even two seemingly unrelated classes can be derived from the base class. Therefore, according to this line of thinking, AAS cannot be negative or 0.
4. $AAS_{i,i} \subseteq \mathbb{R}$, this appears to contradict common sense, as cognition implies that a class should be infinitely similar to itself. This is because even in the same class, different individuals will still present significant differences, which can lead to further subdivision of the class into more sub-classes. In the theory, a class can be subdivided into numerous classes based on various characteristics.

2 Relatively Specific Similarity (RSS)

The RRS should satisfy the following constraints:

1. $C \equiv n$, in specific practical problems, number of classes C should be a constant.
2. $RSS_{i,j} \not\equiv RSS_{j,i}$

   Proof. For presentation brevity, the severity grades of COPD are classified into three categories: gold1, gold2, and gold3. These categories are denoted as c_i, $i \in \{1, 2, 3\}$. Assuming that all individuals within these three classes have an equal level of dissimilarity, then $\gamma(c_1, c_1) = \gamma(c_2, c_2) = \gamma(c_3, c_3)$.

$$RSS_{1,2} = \frac{AAS_{1,2}}{\sum_{k=1}^{3} AAS_{1,k}} = \frac{\gamma(c_1, c_2)}{\sum_{k=1}^{3} \gamma(c_1, c_k)},$$

$$RSS_{2,1} = \frac{\gamma(c_2, c_1)}{\sum_{k=1}^{3} \gamma(c_2, c_k)}.$$

   Obviously, in COPD, the similarity between gold2 and gold3 should be greater than that between gold1 and gold3, so $\gamma(c_1, c_3) < \gamma(c_2, c_3) \longrightarrow RSS_{1,2} > RSS_{2,1}$.

3. $RSS > 0$, RSS can be derived from AAS, so it maintains the same positive and negative properties as AAS.
4. $RSS_{i,i} \subseteq \mathbb{R}$, as with AAS, RSS should not be infinite even within a specific problem because of the intra-class differrence.